P150

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots\ldots$$

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Question 3 continued

Q36.

$$(2x+3)^{-1} = 3^{-1}\left(1+\frac{2x}{3}\right)^{-1}$$

$$= \frac{1}{3}\left[1 - \frac{2x}{3} + \frac{(-1)(-2)}{2\times 1}\left(\frac{2x}{3}\right)^2 + \frac{(-1)(-2)(-3)}{3\times 2\times 1}\left(\frac{2x}{3}\right)^3 + \ldots\right.$$

$$= \frac{1}{3}\left(1 - \frac{2}{3}x + \frac{4}{9}x^2 - \frac{8}{27}x^3 + \ldots\right)$$

$$= \left(\frac{1}{3} - \frac{2}{9}x + \frac{4}{27}x^2 - \frac{8}{81}x^3 + \ldots\right)$$

(b) $$\frac{\sin 2x}{3+2x} = \sin 2x\,(3+2x)^{-1}$$

$$= \left(2x - \frac{(2x)^3}{3!} + \ldots\right)\left(1 - \frac{2}{9}x + \frac{4}{27}x^2 - \frac{8}{81}x^3 + \ldots\right)$$

$$\frac{2}{3}x - \frac{4}{9}x^2 + \left(\frac{8}{27} - \frac{4}{9}\right)x^3 + \left(\frac{8}{27} - \frac{16}{81}\right)x^4 + \ldots$$

$$= \frac{2}{3}x - \frac{4}{9}x^2 - \frac{4}{27}x^3 + \frac{8}{81}x^4 + \ldots\ldots$$

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Question 2 continued

Q37

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots = 1 + \left(-\frac{x^2}{2} + \frac{x^4}{24}\right)$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} + \dots$$

$$\ln\left[1 + \left(-\frac{x^2}{2} + \frac{x^4}{24}\right)\right] = \left(-\frac{x^2}{2} + \frac{x^4}{24}\right) - \frac{1}{2}\left(-\frac{x^2}{2} + \frac{x^4}{24}\right)^2 + \dots$$

$$= -\frac{x^2}{2} - \frac{x^4}{12} - \dots$$

b) $\ln(\sec x) = \ln\left(\frac{1}{\cos x}\right) = -\ln\cos x$

$$= -\left(-\frac{x^2}{2} - \frac{x^4}{12} - \dots\right) = \frac{x^2}{2} + \frac{x^4}{12} + \dots$$

Q2

(Total 7 marks)

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P147

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Question 1 continued

Q20

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 65\sin 2x$$

Complementary function:

Auxilliary Equation:

$m^2 + 4m + 5 = 0$

$(m+2)^2 = -1$

$\underline{m = -2 \pm i}$

C.F: $y = e^{-2x}(A\cos x + B\sin x)$.

Particular integral

let $y = \lambda\cos\omega x + \mu\sin\omega x$

$y = \lambda\cos 2x + \mu\sin 2x$

$\frac{dy}{dx} = -2\lambda\sin 2x + 2\mu\cos 2x$

$\frac{d^2y}{dx^2} = -4\lambda\cos 2x - 4\mu\sin 2x$

$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 65\sin 2x$

$(-4\lambda\cos 2x - 4\mu\sin 2x) + 4(-2\lambda\sin 2x + 2\mu\cos 2x) + 5(\lambda\cos 2x + \mu\sin 2x) = 65\sin 2x$

Compare coefficients of $\sin 2x$ and $\cos 2x$.

$\lambda = -8$ and $\mu = 1$

$y = e^{-2x}(A\cos x + B\sin x) + \sin 2x - 8\cos 2x$

(Total 9 marks)

Q1

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Question 8 continued

b)

$$y = e^{-2x}(A\cos x + B\sin x) + \sin 2x - 8\cos 2x$$

when $x \to 0$, $e^{-2x} \to 0$ and $y \to \sin 2x - 8\cos 2x$

$\sin 2x - 8\cos 2x = R\sin(2x - \alpha)$

$\sin 2x - 8\cos 2x = R\sin 2x\cos\alpha - R\cos 2x\sin\alpha$

$R^2 = 1^2 + 8^2 = 65$

$R = \sqrt{65}$

$\tan\alpha = 8 \Rightarrow \alpha = 82.9°$

$\sqrt{65}\sin(2x - \alpha) \Rightarrow \sqrt{65}\sin(2x - 82.9°)$

Q8

(Total 10 marks)

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Question 9 continued

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 2y = 2e^{-t}.$$

Q21

Complementary function:

Auxiliary Equation:

$m^2 + 2m + 2 = 0$

$(m+1)^2 = -1$

$m = -1 \pm i$

C.F: $y = e^{-t}(A\cos t + B\sin t)$

Particular Integral:

Let $y = ke^{-t}.$

$\frac{dy}{dt} = -ke^{-t}$ and $\frac{d^2y}{dt^2} = ke^{-t}.$

$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 2y = 2e^{-t}.$

$ke^{-t} + 2(-ke^{-t}) + 2ke^{-t} = 2e^{-t}.$

$k = 2$

P.I $= 2e^{-t}.$

$y = e^{-t}(A\cos t + B\sin t) + 2e^{-t}$

b) $y = 1$ and $\frac{dy}{dt} = 1$ at $t = 0$

use above conditions to solve:

$B = 2$ and $A = -1$

$y = e^{-t}(2\sin t - \cos t) + 2e^{-t}.$

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P145

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Question 9 continued

$\frac{dy}{dx} + Py = Q$

Q8

$$t\frac{dv}{dt} - v = t$$

$$\Downarrow$$

$$\frac{dv}{dt} - \frac{1}{t}v = 1$$

$$e^{\int P\,dx} = e^{-\int \frac{1}{t}dt} = e^{-\ln t} = e^{\ln 1/t} = \frac{1}{t}$$

$$\frac{1}{t}\cdot\frac{dv}{dt} - \frac{1}{t}\cdot\frac{1}{t}v = \frac{1}{t}$$

$$\frac{1}{t}\frac{dv}{dt} - \frac{1}{t^2}v = \frac{1}{t}$$

$$\frac{d}{dt}\left(\frac{v}{t}\right) = \frac{1}{t}$$

$$\frac{v}{t} = \int \frac{1}{t}\,dt$$

$$\frac{v}{t} = \ln t + c$$

$$v = t(\ln t + c)$$

(b) When $t = 2$ $v = 3\ ms^{-1}$

$$v = t(\ln t + c) \Rightarrow c = \frac{3}{2} - \ln 2$$

$$v = t\left(\ln t + \frac{3}{2} - \ln 2\right)$$

when $t = 4$ $\Rightarrow$ $v = \underline{8.77\ ms^{-1}}$

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Question 9 continued

Q9

$$y = vx$$

$$\frac{dy}{dx} = \frac{(4x+y)(x+y)}{x^2}$$

$$y = vx$$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\left(v + x\frac{dv}{dx}\right) = \frac{(4x+vx)(x+vx)}{x^2}$$

$$= \frac{4x^2 + 4vx^2 + vx^2 + v^2x^2}{x^2} = 4 + 5v + v^2$$

$$\left(v + x\frac{dv}{dx}\right) = 4 + 5v + v^2$$

$$x\frac{dv}{dx} = v^2 + 4v + 4 = (2+v)^2$$

$$x\frac{dv}{dx} = (v+2)^2$$

b)

$$\int \frac{1}{(2+v)^2}\,dv = \int \frac{1}{x}\,dx$$

$$-\frac{1}{2+v} = \ln x + c$$

$$2 + v = -\frac{1}{\ln x + c}$$

$$v = -2 - \frac{1}{\ln x + c}$$

c) $y = vx \qquad v = \frac{y}{x}$

$$\frac{y}{x} = -2 - \frac{1}{\ln x + c}$$

$$y = -2x - \frac{x}{\ln x + c}$$

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