Q1 June 2009 6668

$$\frac{1}{r(r+2)} = \frac{A}{r} + \frac{B}{r+2} \Rightarrow 1 = A(r+2) + Br$$

$r = 0 \Rightarrow A = \frac{1}{2}$

$r = -2 \Rightarrow B = -\frac{1}{2}$

$$\frac{1}{r(r+2)} = \frac{1}{2r} - \frac{1}{2(r+2)}$$

(b) $$\sum_{r=1}^{n} \frac{4}{r(r+2)} = 4\sum_{r=1}^{n} \frac{1}{r(r+2)}$$

$$\sum_{r=1}^{n} \frac{1}{r(r+2)} = \sum_{r=1}^{n} \left(\frac{1}{2r} - \frac{1}{2(r+2)} \right)$$

$r=1$ $\left(\frac{1}{2} - \frac{1}{6}\right)$

$r=2$ $\left(\frac{1}{4} - \frac{1}{8}\right)$

$r=3$ $\left(\frac{1}{6} - \frac{1}{10}\right)$

$r=4$ $\left(\frac{1}{8} - \frac{1}{12}\right)$

$r=5$ $\left(\frac{1}{10} - \frac{1}{14}\right)$

$r=n-1$ $\left(\frac{1}{2(n-1)} - \frac{1}{2(n+1)}\right)$

$r=n$ $\left(\frac{1}{2n} - \frac{1}{2(n+2)}\right)$

$$\frac{1}{2} + \frac{1}{4} - \frac{1}{2(n+1)} - \frac{1}{2(n+2)}$$

$$\frac{3}{4} - \frac{1}{2(n+1)} - \frac{1}{2(n+2)}$$

$$4\sum_{r=1}^{n} \frac{1}{r(r+2)} = 4 * \left[\frac{3}{4} - \frac{1}{2(n+1)} - \frac{1}{2(n+2)} \right]$$

$$= 4 \times \left[\frac{3(n+1)(n+2) - 2(n+2) - 2(n+1)}{4(n+1)(n+2)} \right]$$

$$= \frac{3n^2 + 9n + 6 - 2n - 4 - 2n - 2}{(n+1)(n+2)} = \frac{n(3n+5)}{(n+1)(n+2)}$$

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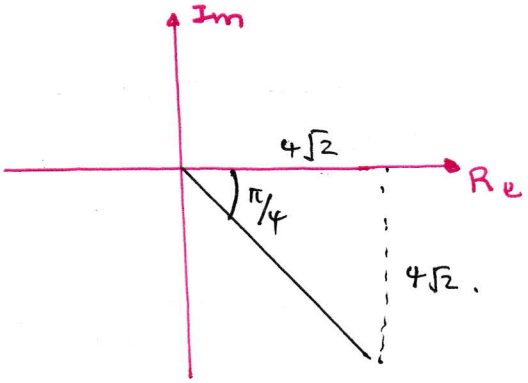
$Q_2 \quad z^3 = 4\sqrt{2} - 4\sqrt{2}\,i$



$$r = \sqrt{(4\sqrt{2})^2 + (4\sqrt{2})^2}$$

$$= \underline{8}$$

$$\text{Arg}(z^3) = -\pi/4$$

$$z^3 = 8\left(\cos(-\pi/4) + i\sin(-\pi/4)\right)$$

$$z = 2\left[\cos\left(-\frac{\pi}{4} + 2k\pi\right) + i\sin\left(-\frac{\pi}{4} + 2k\pi\right)\right]^{1/3}$$

$$z = 2\left[\cos\left(\frac{8k-1}{12}\right)\pi + i\sin\left(\frac{8k-1}{12}\right)\pi\right]$$

$$k = -1 \Rightarrow z = 2\left[\cos\left(-\frac{9\pi}{12}\right) + i\sin\left(-\frac{9\pi}{12}\right)\right]$$

$$k = 0 \Rightarrow z = 2\left[\cos\left(-\frac{\pi}{12}\right) + i\sin\left(-\frac{\pi}{12}\right)\right]$$

$$k = 1 \Rightarrow z = 2\left[\cos\frac{7\pi}{12} + i\sin\frac{7\pi}{12}\right]$$

$Q_3 \quad \sin x \frac{dy}{dx} - y\cos x = \sin 2x \sin x \Rightarrow \frac{dy}{dx} - \frac{\cos x}{\sin x} y = \sin 2x$

$$e^{\int P dx} = e^{-\int \frac{\cos x}{\sin x} dx} = e^{-\ln|\sin x|} = \text{Cosec}\,x$$

$$\text{Cosec}\,x \frac{dy}{dx} - \text{cosec}\,x \cdot \frac{\cos x}{\sin x} y = \sin 2x \,\text{Cosec}\,x.$$

$$\frac{d}{dx}(y\,\text{cosec}\,x) = \frac{2\sin x \cos x}{\sin x} \Rightarrow y\,\text{cosec}\,x = 2\int \cos x\,dx$$

$$= 2\sin x + c$$

$$y = \underline{\frac{2\sin x + c}{\text{Cosec}\,x}}$$

Q4

$$\text{Area} = 2 \times \frac{1}{2}\int_0^{\pi} (a + 3\cos\theta)^2 \, d\theta$$

$$= \int_0^{\pi} (9\cos^2\theta + 6a\cos\theta + a^2)\, d\theta.$$

$\because \cos^2 A \equiv \frac{\cos 2A + 1}{2}$.

$$= \int_0^{\pi} 9\left(\frac{\cos 2\theta + 1}{2}\right) + 6a\cos\theta + a^2)\, d\theta.$$

$$= \left[\frac{9}{4}\sin 2\theta + 6a\sin\theta + \left(a^2 + \frac{9}{2}\right)\theta\right]_0^{\pi} = \pi a^2 + \frac{9\pi}{2}$$

$$\therefore \pi a^2 + \frac{9\pi}{2} = \frac{107}{2}\pi \Rightarrow a^2 = 49$$

$$a = 7$$

Q5. $y = \sec^2 x \Rightarrow \frac{dy}{dx} = (2\sec x)\cdot \sec x\tan x = 2\sec^2 x\tan x$

$$\frac{d^2y}{dx^2} = (4\sec x)\cdot \sec x\tan^2 x + 2\sec^4 x = 4\sec^2 x\tan^2 x + 2\sec^4 x$$

$$4\sec^2 x(\sec^2 x - 1) + 2\sec^4 x$$

$$= 6\sec^4 x - 4\sec^2 x$$

(b) $y = \sec^2 x \Rightarrow f(\pi/4) = 2$

$f'(\pi/4) = 4 \quad f''(\pi/4) = 16$

$$\frac{d^3y}{dx^3} = 24\sec^3 x(\sec x\tan x) - 8\sec x(\sec x\tan x)$$

$f'''(\pi/4) = 80$

$$f(x) = 2 + 4(x - \pi/4) + 8(x - \pi/4)^2 + \frac{40}{3}(x - \pi/4)^3$$

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Q6

$$w = \frac{z}{z+i} \Rightarrow wz + wi = z$$

$$z = \frac{iw}{1-w}$$

$$|z| = 3 \Rightarrow \left|\frac{iw}{1-w}\right| = 3$$

$$|wi| = 3|1-w| \Rightarrow |w| = 3|w-1|$$

$$w = u + iv$$

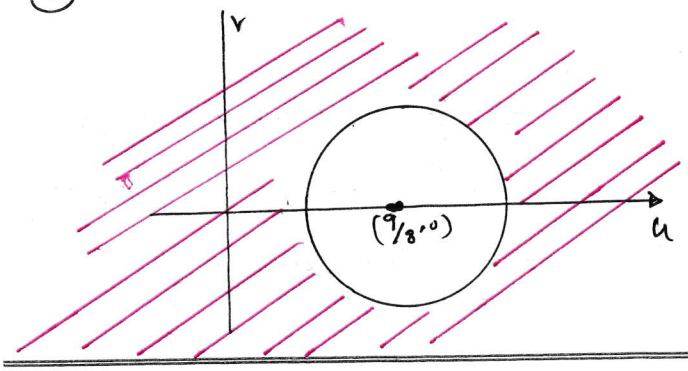
$$u^2+v^2 = 9[(u-1)^2+v^2]$$

$$u^2+v^2 = 9u^2 - 18u + 9 + 9v^2$$

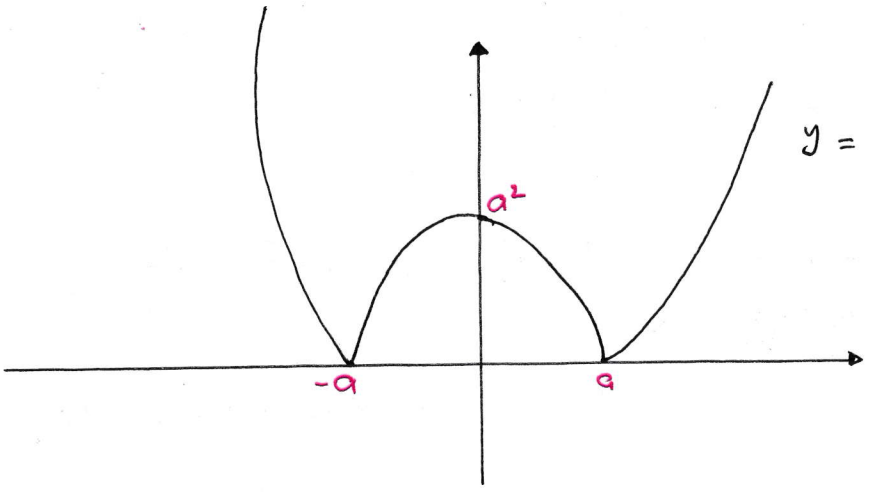
$$u^2 - \frac{9}{4}u + v^2 = -\frac{9}{8}$$

$$\left(u - \frac{9}{8}\right)^2 + v^2 = \frac{9}{64} \Rightarrow \text{Centre } (9/8, 0) \quad r = 3/8$$

(b)



Q7.



$y = |x^2 - a^2|$.

⑤ $|x^2-a^2| = a^2-x$

$x^2-a^2 = a^2-x$ OR $x^2-a^2 = x-a^2$

$x^2+x-2a^2=0$ $x^2-x=0$

$\left(x+\frac{1}{2}\right)^2 = 2a^2+\frac{1}{4}$ $x=0$ or 1

$x = -\frac{1}{2} \pm \sqrt{2a^2+\frac{1}{4}}$

⑥ $x < \frac{-1-\sqrt{1+8a^2}}{2}$ OR $x > \frac{-1+\sqrt{1+8a^2}}{2}$ OR $0<x<1$

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⑧ $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 2e^{-t}$

$m^2+5m+6=0 \Rightarrow m=-3$ or -2

$y = Ae^{-3t} + Be^{-2t}$

Particular Integral

Let $x = \lambda e^{-t}$.

$\frac{dx}{dt} = -\lambda e^{-t}$

$\frac{d^2x}{dt^2} = \lambda e^{-t}$.

$\lambda e^{-t} + 5(-\lambda e^{-t}) + 6\lambda e^{-t} = 2e^{-t}$.

$\Rightarrow \lambda = 1$

$x = Ae^{-3t} + Be^{-2t} + e^{-t}$.

$x=0$ at $t=0 \Rightarrow A+B=-1$

$x=0$, $\frac{dx}{dt}=2$ at $t=0 \Rightarrow 2 = -3A-2B-1$ $\Rightarrow$ $A=-1$, $B=0$

$\therefore x = -e^{-3t} + e^{-t}$

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(b) $x = -e^{-3t} + e^{-t} \Rightarrow \frac{dx}{dt} = 3e^{-3t} - e^{-t}$

maximum distance at $\frac{dx}{dt} = 0 \Rightarrow 3e^{-3t} - e^{-t} = 0$

$3 - e^{2t} = 0$

$e^{2t} = 3$

$t = \frac{1}{2}\ln 3$

$x_{max} = -e^{-\frac{3}{2}\ln 3} + e^{-\frac{1}{2}\ln 3}$

$x = -\frac{1}{3\sqrt{3}} + \frac{1}{\sqrt{3}} = \frac{2}{3\sqrt{3}} = \frac{2\sqrt{3}}{9}$

$\frac{d^2x}{dt^2} = -9e^{-3t} + e^{-t}$ $\quad t = \frac{1}{2}\ln 3$

$\frac{d^2x}{dt^2} = -9e^{-\frac{3}{2}\ln 3} + e^{-\frac{1}{2}\ln 3}$

$= -\frac{9}{3\sqrt{3}} + \frac{1}{\sqrt{3}} = -\frac{2}{\sqrt{3}}$

since $\frac{d^2y}{dx^2} < 0$ x is the maximum distance.