Write your name here

13 May 2019

Surname	Other names
FOWAI	ISSA

Pearson Edexcel
International Advanced Level

Centre Number	Candidate Number

Further Pure Mathematics F3

Advanced/Advanced Subsidiary

Monday 25 June 2018 – Morning
Time: 1 hour 30 minutes

Paper Reference
WFM03/01

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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1. Solve the equation

$$15\,\text{sech}^2 x + 7\tanh x = 13$$

Give your answers in terms of simplified natural logarithms.

(6)

$\text{sech}^2 x = 1 - \tanh^2 x$

$15\,\text{sech}^2 x + 7\tanh x = 13$

$15(1 - \tanh^2 x) + 7\tanh x = 13$

$15 - 15\tanh^2 x + 7\tanh x = 13$

$15\tanh^2 x - 7\tanh x - 2 = 0$

$(5\tanh x + 1)(3\tanh x - 2) = 0$

$\tanh x = -\frac{1}{5}$ OR $\frac{2}{3}$

$x = \text{artanh}\left(-\frac{1}{5}\right)$ OR $\text{artanh}\,\frac{2}{3}$

$x = \frac{1}{2}\ln\frac{2}{3}$ OR $\frac{1}{2}\ln 5$

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2.

$$\mathbf{A} = \begin{pmatrix} 3 & 2 \\ 2 & 6 \end{pmatrix}$$

(a) Find the eigenvalues and corresponding normalised eigenvectors of the matrix **A**.

(7)

(b) Write down a matrix **P** and a diagonal matrix **D** such that $\mathbf{P}^{\mathrm{T}}\mathbf{AP} = \mathbf{D}$.

(2)

$$A = \begin{pmatrix} 3 & 2 \\ 2 & 6 \end{pmatrix}$$

$$(A - \lambda I) = \begin{pmatrix} 3-\lambda & 2 \\ 2 & 6-\lambda \end{pmatrix}$$

$$\det(A - \lambda I) = 0$$

$$(3-\lambda)(6-\lambda) - 4 = 0$$

$$18 - 3\lambda - 6\lambda + \lambda^2 - 4 = 0$$

$$\lambda^2 - 9\lambda + 14 = 0$$

$$\lambda = 2 \text{ or } 7$$

(a)

$$\begin{pmatrix} 3 & 2 \\ 2 & 6 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 2\begin{pmatrix} x \\ y \end{pmatrix}$$

$$3x + 2y = 2x \implies x = -2y$$

$$2x + 6y = 2y \implies$$

$$\Downarrow$$

$$4y = -2x$$

$$y = -x/2.$$

Let $x = 2$, $y = -1$ $\quad \begin{pmatrix} 2 \\ -1 \end{pmatrix}$

$$\begin{pmatrix} \frac{2}{\sqrt{5}} \\ -\frac{1}{\sqrt{5}} \end{pmatrix}$$

P 5 1 5 1 5 A 0 4 3 2

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Question 2 continued

$$\begin{pmatrix} 3 & 2 \\ 2 & 6 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 7\begin{pmatrix} x \\ y \end{pmatrix}$$

$3x + 2y = 7x \Rightarrow$

$2x + 6y = 7y \Rightarrow y = 2x$

let $x = 1$ $\quad y = 2$.

$\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ $\qquad \begin{pmatrix} \frac{1}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} \end{pmatrix}$ ← normalised vectors

$\begin{pmatrix} \frac{1}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} \end{pmatrix}$ and $\begin{pmatrix} \frac{2}{\sqrt{5}} \\ \frac{-1}{\sqrt{5}} \end{pmatrix}$

(b) $D = \begin{pmatrix} 7 & 0 \\ 0 & 2 \end{pmatrix}$ $\qquad P = \begin{pmatrix} \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} & -\frac{1}{\sqrt{5}} \end{pmatrix}$

OR

$D = \begin{pmatrix} 0 & 7 \\ 2 & 0 \end{pmatrix}$ $\qquad P = \begin{pmatrix} \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \\ -\frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \end{pmatrix}$.

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3. Given that

$$y = \arctan\left(\frac{\sin x}{\cos x - 1}\right) \qquad x \neq 2n\pi, \qquad n \in \mathbb{Z}$$

Show that

$$\frac{dy}{dx} = k$$

where k is a constant to be found.

(6)

$$y = \arctan\left(\frac{\sin x}{\cos x - 1}\right) = \frac{\pi}{2}$$

$$\frac{d}{dx}\left(\frac{\sin x}{\cos x - 1}\right) = \frac{\cos x(\cos x - 1) + \sin^2 x}{(\cos x - 1)^2}$$

$$\frac{dy}{dx} = \frac{1}{1 + \left(\frac{\sin x}{\cos x - 1}\right)^2} \cdot \left(\frac{\cos x(\cos x - 1) + \sin^2 x}{(\cos x - 1)^2}\right)$$

$$\frac{dy}{dx} = \frac{(\cos x - 1)^2}{(\cos x - 1)^2 + \sin^2 x}\left(\frac{1 - \cos x}{(\cos x - 1)^2}\right)$$

$$\frac{dy}{dx} = \frac{1 - \cos x}{(\cos^2 x - 2\cos x + 1) + \sin^2 x}$$

$$\frac{dy}{dx} = \frac{1}{2}$$

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4. The hyperbola H has equation

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

The line l is a normal to H at the point $P(a\sec\theta, b\tan\theta)$, $0 < \theta < \frac{\pi}{2}$

(a) Using calculus, show that an equation for l is

$$ax\sin\theta + by = (a^2 + b^2)\tan\theta$$

(5)

The line l meets the x-axis at the point Q, and the point M is the midpoint of PQ.

(b) Find the coordinates of M.

(3)

(c) Hence find the cartesian equation of the locus of M as θ varies, giving your answer in the form $y^2 = f(x)$.

(4)

$\frac{dy}{dx} = \frac{b\sec^2\theta}{a\sec\theta\tan\theta}$,

$m_N = -\frac{a\sec\theta\tan\theta}{b\sec^2\theta} = -\frac{a}{b}\sin\theta$

$y - b\tan\theta = -\frac{a}{b}\sin\theta(x - a\sec\theta)$

$by - b^2\tan\theta = -ax\sin\theta + a^2\tan\theta$

$ax\sin\theta + by = a^2\tan\theta + b^2\tan\theta$

$ax\sin\theta + by = (a^2 + b^2)\tan\theta$

(b) At Q $y = 0$

$ax\sin\theta = (a^2 + b^2)\tan\theta$.

$x = \frac{(a^2+b^2)\tan\theta}{a\sin\theta} = \frac{(a^2+b^2)}{a}\sec\theta$.

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Question 4 continued

$$M = \left(\frac{1}{2} \left(\frac{(a^2+b^2)\sec\theta}{a} + a\sec\theta \right), \frac{b\tan\theta}{2} \right)$$

$$M = \left(\frac{(2a^2+b^2)}{2a}\sec\theta, \frac{b}{2}\tan\theta \right)$$

c.

$$x = \frac{(2a^2+b^2)\sec\theta}{2a} \qquad \sec\theta = \frac{2ax}{2a^2+b^2}$$

$$y = \frac{b}{2}\tan\theta \Rightarrow \tan\theta = \frac{2y}{b}$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \left(\frac{2y}{b}\right)^2 = \left(\frac{2ax}{2a^2+b^2}\right)^2$$

$$y^2 = \frac{b^2}{4}\left(\frac{4a^2x^2}{(2a^2+b^2)^2} - 1 \right)$$

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5.

$$\mathbf{M} = \begin{pmatrix} 4 & -5 & 0 \\ k & 2 & 0 \\ -3 & -5 & k \end{pmatrix}, \text{ where } k \text{ is a real constant, } k \neq 0,\ k \neq -\frac{8}{5}$$

(a) Find, in terms of k, the inverse of the matrix $\mathbf{M}$.

(5)

A transformation $T: \mathbb{R}^3 \to \mathbb{R}^3$ is represented by the matrix

$$\begin{pmatrix} 4 & -5 & 0 \\ -1 & 2 & 0 \\ -3 & -5 & -1 \end{pmatrix}$$

The transformation T maps the plane Π_1 onto the plane Π_2

Given that the plane Π_2 has equation $2x - z = 4$

(b) find a cartesian equation of the plane Π_1

(6)

Minors : $\begin{pmatrix} 2k & k^2 & -5k+6 \\ -5k & 4k & -35 \\ 0 & 0 & 8+5k \end{pmatrix}$ $\det(M) = 5k^2 + 8k$

cofactors = $\begin{pmatrix} 2k & -k^2 & 6-5k \\ 5k & 4k & 35 \\ 0 & 0 & 8+5k \end{pmatrix}$

$M^{-1} = \frac{1}{5k^2+8k} \begin{pmatrix} 2k & 5k & 0 \\ -k^2 & 4k & 0 \\ 6-5k & 35 & 8+5k \end{pmatrix}$

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Π_2 : $x = s$ $\quad y = t$ $\quad z = 2s - 4$

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Question 5 continued

$$\underset{M}{\begin{pmatrix} 4 & -5 & 0 \\ -1 & 2 & 0 \\ -3 & -5 & -1 \end{pmatrix}} \underset{x}{\begin{pmatrix} x \\ y \\ z \end{pmatrix}} = \begin{pmatrix} 4x - 5y \\ -x + 2y \\ -3x - 5y - z \end{pmatrix}$$

$$\begin{pmatrix} 4x - 5y \\ -x + 2y \\ -3x - 5y - z \end{pmatrix} = \begin{pmatrix} s \\ t \\ 2s - 4 \end{pmatrix}$$

$11x - 5y + z = 4$

$$\begin{pmatrix} 4 & -5 & 0 \\ -1 & 2 & 0 \\ -3 & -5 & -1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{matrix} x \\ y \\ z \end{matrix}$$

↳ any general point (a, b, c) on first plane

$4a - 5b = x$

$-a + 2b = y$

$-3a - 5b - c = z$.

$2x - z = 4$

$2(4a - 5b) - (-3a - 5b - c) = 4$

$8a - 10b + 3a + 5b + c = 4$

$11a - 5b + c = 4$

$11x - 5y + z = 4$

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6. The curve C has parametric equations

$$x = \theta - \tanh\theta, \quad y = \operatorname{sech}\theta, \quad 0 \leqslant \theta \leqslant \ln 3$$

(a) Find

(i) $\dfrac{dx}{d\theta}$ $= 1 - \operatorname{sech}^2\theta.$

(ii) $\dfrac{dy}{d\theta}$ $= -\operatorname{sech}\theta\tanh\theta.$

(2)

The curve C is rotated through 2π radians about the x-axis.

(b) Find the exact area of the curved surface formed, giving your answer as a multiple of π.

(5)

(b) $S = 2\pi\int \operatorname{sech}\theta\sqrt{(1-\operatorname{sech}^2\theta)^2 + (-\operatorname{sech}\theta\tanh\theta)^2}\,d\theta.$

$S = 2\pi\int \operatorname{sech}\theta\sqrt{1-\operatorname{sech}^2\theta}\,d\theta$

$S = 2\pi\int \operatorname{sech}\theta\cdot\tanh\theta\cdot d\theta$

$S = 2\pi\left[-\operatorname{sech}\theta\right]$

$S = 2\pi\left[-\operatorname{sech}\theta\right]_0^{\ln 3} = 0.8\pi$

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7. The plane Π_1 has equation $x + y + z = 3$ and the plane Π_2 has equation $2x + 3y - z = 4$

The planes Π_1 and Π_2 intersect in the line L.

(a) Find a cartesian equation for the line L.

(6)

The plane Π_3 has equation

$$\mathbf{r} \cdot \begin{pmatrix} 5 \\ -4 \\ 4 \end{pmatrix} = 12$$

The line L meets the plane Π_3 at the point A.

(b) Find the coordinates of A.

(3)

(c) Find the acute angle between $\overrightarrow{OA}$ and the line L, where O is the origin.
Give your answer in degrees to one decimal place.

(3)

Π_1 : $x + y + z = 3$

Π_2 : $2x + 3y - z = 4$

$$\mathbf{n} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 1 \\ 2 & 3 & -1 \end{vmatrix} = \begin{pmatrix} -4 \\ 3 \\ 1 \end{pmatrix}$$

Let $x = 0$ $\quad y + z = 3$ $\quad 3y - z = 4$ $\Rightarrow$ $y = 7/4$, $z = 5/4$

$$\frac{x}{-4} = \frac{y - 7/4}{3} = \frac{z - 5/4}{1} = \lambda$$

if $x = 1$ $\quad (1, 1, 1)$

$$\frac{x-1}{-4} = \frac{y-1}{3} = \frac{z-1}{1} = \lambda$$

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Question 7 continued

(b)

π_2: $5x - 4y + 4z = 12$

$5(-4\lambda) - 4(\frac{7}{4} + 3\lambda) + 4(\frac{5}{4} + \lambda) = 12$

$-20\lambda - 7 - 12\lambda + 5 + 4\lambda = 12$

$-28\lambda = 14$

$\lambda = -\frac{1}{2}$

A $(2, \frac{1}{4}, \frac{3}{4})$ ans

(c)

$$\cos\theta = \frac{\begin{pmatrix} 2 \\ 1/4 \\ 3/4 \end{pmatrix} \cdot \begin{pmatrix} -4 \\ 3 \\ 1 \end{pmatrix}}{\sqrt{2^2 + (\frac{1}{4})^2 + (\frac{3}{4})^2}\sqrt{4^2 + 3^2 + 1^2}} = \text{no}$$

$$\cos\theta = \frac{-13/2}{\sqrt{\frac{37}{8}} \cdot \sqrt{26}}$$

$\theta = 53.6°$

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8.

$$I_n = \int \frac{x^n}{\sqrt{(x^2 + k^2)}} \, dx \qquad \text{where } k \text{ is a constant and } n \in \mathbb{Z}^+$$

(a) Show that, for $n \geqslant 2$

$$I_n = \frac{x^{n-1}}{n}(x^2 + k^2)^{\frac{1}{2}} - \frac{(n-1)}{n}k^2 I_{n-2}$$

(7)

(b) Hence find the exact value of

$$\int_0^1 \frac{x^5}{\sqrt{(x^2 + 1)}} \, dx$$

(5)

$I_n = \int \frac{x^n}{\sqrt{x^2+k^2}} \, dx = \int \left(x^{n-1} \cdot \frac{x}{\sqrt{x^2+k^2}} \right) dx$

Let $u = x^{n-1}$ $\qquad v' = \frac{x}{\sqrt{x^2+k^2}}$

$u' = n-1 \, x^{n-2}$ $\qquad v = (x^2+k^2)^{1/2}$

$I_n = x^{n-1}(x^2+k^2)^{1/2} - \int (n-1) x^{n-2} (x^2+k^2)^{1/2} \, dx$

$I_n = x^{n-1}(x^2+k^2)^{1/2} - (n-1) \int x^{n-2} \frac{(x^2+k^2)}{\sqrt{x^2+k^2}} \, dx$

$I_n = x^{n-1}(x^2+k^2)^{1/2} - (n-1) \int \frac{x^n}{\sqrt{x^2+k^2}} \, dx - (n-1) \int \frac{k^2 x^{n-2}}{\sqrt{x^2+k^2}} \, dx$

$I_n = x^{n-1}(x^2+k^2)^{1/2} - (n-1) I_n - (n-1) k^2 I_{n-2}$

$n I_n = x^{n-1}(x^2+k^2)^{1/2} - (n-1) k^2 I_{n-2}$

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Question 8 continued

$$I_n = \frac{x^{n-1}}{n}\left(k^2+x^2\right)^{1/2} - \frac{(n-1)}{n}k^2 I_{n-2}^{*}$$

$$I_5 = \int_0^1 \frac{x^5}{\sqrt{x^2+1}}\,dx$$

$$I_5 = \frac{x^4}{5}\left(x^2+1\right)^{1/2} - \tfrac{4}{5}I_3$$

$$I_3 = \frac{x^2}{3}\left(x^2+1\right)^{1/2} - \tfrac{2}{3}I_1$$

$$I_1 = \int_0^1 \frac{x}{\sqrt{x^2+1}}\,dx = \left[\sqrt{x^2+1}\right]_0^1 = \sqrt{2}$$

$$I_5 = \frac{7}{15}\sqrt{2} - \frac{8}{15}$$

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